Updated predictions for the total production cross sections of top and of heavier quark pairs at the Tevatron and at the LHC

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Abstract

We present updated predictions for the total production cross section of top-quark pairs at the Tevatron and at the LHC, and, at the LHC, of heavy-quark pairs with mass in the range $0.5 - 2$ TeV. For $t\bar{t}$ production at the LHC we also present results at $\sqrt{S} = 10$ TeV, in view of the expected accelerator conditions during the forthcoming 2008 run. Our results are accurate at the level of next-to-leading order in $\alpha_{\rm s}$, and of next-to-leading threshold logarithms (NLO+NLL). We adopt the most recent parametrizations of parton distribution functions, and compute the corresponding uncertainties. We study the dependence of the results on the top mass, and we assess the impact of missing higher-order corrections by independent variations of factorisation and renormalisation scales.

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1 Top production

One of the first tasks of the Large Hadron Collider (LHC) experiments when, later in 2008, they will start taking data, will be to re-discover the Standard Model. This will serve the double purpose of making sure that the detectors are well understood and work properly, as well as of improving the precision of previous measurements. With a predicted cross section for top quark pairs about a hundred times larger than at the Fermilab Tevatron, and a much higher design luminosity, the LHC is poised to become a real "top factory" [1]. This will allow for better measurements of the mass and the cross section. The former is expected to be measured with an ultimate uncertainty below 1 GeV [1, 2] (to be compared with the most recent determination from the Tevatron, $m_t = 172.6 \pm 0.8 \pm 1.1$ GeV [3]). The cross section is expected to be measured within a year with a 15% accuracy, and eventually with an accuracy probably limited only by the knowledge of the LHC luminosity, expected to reach a precision of a few per cent [4].

Experimental measurements of the total cross section at the Tevatron [5, 6] are usually compared to predictions [8, 9] compiled a few years ago[1]. These predictions made use of the next-to-leading order (NLO) calculations of [11], and of the soft-gluon next-to-leading-log (NLL) threshold resummation results obtained in [12, 13] and [14] respectively. Some logarithmic contributions of order higher than NLL have been included in the results of ref. [9, 15] and subsequent papers. In this way, while a complete NNLO calculation is still unavailable (the first ingredients, two loop virtual corrections in the ultrarelativistic limit [16, 17], fully massive results for the two-loop contribution in the quark-quark channel [18, 19] and for the one-loop-squared contributions [20, 21], real emission at one loop [22], have been recently obtained), *some* NNLO contributions of order α_{s}^4, of soft origin, can be obtained through an expansion and truncation of the Sudakov exponent. More recently, while this paper was being completed, the final ingredients required for a complete resummation of soft-gluon next-to-next-to-leading logarithms (NNLL) have been calculated [23], and the relative impact on the total cross sections was explored.

A key feature of our most recent study in [8] was an extensive exploration of the theoretical uncertainties affecting the prediction. The effect of the independent variations of renormalisation and of factorisation scales, which is the customary way to assess the impact of unknown higher-order contributions, was explored in detail. Parton distribution function sets (PDFs) providing a mean to estimate the associated uncertainty [24] were also used. It was determined in [8] that a significant fraction of the overall $\pm 10 - 13\%$ uncertainty in $p\bar{p}$ production at the Tevatron was originating from the PDFs, though higher orders were also contributing a fair share. This result should now be revisited on a number of counts. First, new PDF sets with errors, CTEQ6.5 [25], MRST2006nnlo [26] and CTEQ 6.6 [27] have appeared in the past few years. It is legitimate to wonder if they might come with a reduced uncertainty. Second, a similarly careful job of estimating the theoretical uncertainties for the best available prediction should be made for the LHC too. Third, since the most recent Tevatron measurements point to a lower mass than the central value $m_t = 175$ GeV used in [8], it is useful to produce numerical predictions for an updated value of the top mass. Note that an analysis of the $t\bar{t}$ cross section has recently been performed in [27]. This study, carried out at the fixed-order, NLO level, focuses on the correlations of the top cross section with other observables, analyzed as a function of the PDF sets.

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[1]The precision of the cross section experimental measurements has recently become sufficiently good that extractions of the top mass by comparing the measured cross section with the calculated value have become possible, and have been performed [10].

We shall present our results in the form

$$\sigma = \sigma(\text{central})^{+\Delta\sigma_{\mu+}}_{-\Delta\sigma_{\mu-}} {}^{+\Delta\sigma_{\text{PDF}+}}_{-\Delta\sigma_{\text{PDF}-}} \,, \tag{1}$$

where $\sigma(\text{central})$ is our best prediction, and $\Delta\sigma_{\mu\pm}$ and $\Delta\sigma_{\text{PDF}\pm}$ quantify the uncertainties due to higher perturbative orders and PDF choices, as specified in what follows.

In order to streamline the calculation of the overall uncertainty (unknown higher orders and PDFs) we modify slightly the method employed in ref. [8] and proceed as follows.

- Our best prediction $\sigma(\text{central})$ is computed by setting the renormalisation and factorisation scales equal to m_t, and with the central PDF set (within a given PDF error family). The cross section is calculated to NLO+NLL accuracy, exactly as in ref. [8].

- The uncertainty on higher orders is estimated by varying the factorisation and the renormalisation scales μ_{F} and μ_{R} independently around a central scale set by the top mass m_t. We define the ratios

$$\xi_{\text{F}} = \mu_{\text{F}}/m_t \,, \qquad \xi_{\text{R}} = \mu_{\text{R}}/m_t \,, \tag{2}$$

and we allow them to vary in the regions $0.5 \leq \xi_{\text{F}}, \xi_{\text{R}} \leq 2$, with the condition that $0.5 \leq \xi_{\text{F}}/\xi_{\text{R}} \leq 2$. This means that none of the ratios μ_{F}/m_t, μ_{R}/m_t and $\mu_{\text{F}}/\mu_{\text{R}}$ can be larger than two or smaller than one-half, in order not to have in the perturbative expansion logarithms of arguments larger than a chosen (admittedly arbitrary) amount. Within this region the NLO+NLL cross section is evaluated[2], and used to compute[3]

$$\Delta\sigma_{\mu+} = \max_{\{\xi_{\text{F}},\xi_{\text{R}}\}} \left[\sigma(\xi_{\text{F}}, \xi_{\text{R}}) - \sigma(1,1) \right] , \tag{3}$$

$$\Delta\sigma_{\mu-} = - \min_{\{\xi_{\text{F}},\xi_{\text{R}}\}} \left[\sigma(\xi_{\text{F}}, \xi_{\text{R}}) - \sigma(1,1) \right] . \tag{4}$$

All cross sections in these formulae are evaluated with the central PDF set (thus, $\sigma(1,1) \equiv \sigma(\text{central})$ here). We also introduce the symbols

$$\text{Scales}+ \;=\; \sigma(\text{central}) + \Delta\sigma_{\mu+} \tag{5}$$

$$\text{Scales}- \;=\; \sigma(\text{central}) - \Delta\sigma_{\mu-} \tag{6}$$

which we shall use in the following.

By doing so we have established a variation interval of the cross section that can be considered as a reasonable estimate of the uncertainty due to unknown higher orders. It should be noted, however, that such an uncertainty should by no means be considered as distributed according to some probability law (for instance, with a Gaussian distribution) around the central value. In fact, it is more similar to a systematic than to a statistical uncertainty. This means that further arbitrary choices will have to be made in order to assign a 'confidence level' to this interval.

- Modern PDF sets come with a procedure to evaluate the propagation of their uncertainty onto a given physical observable. This is done by exploring the effect of using, along with a

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[2] A NLL resummation function with independent renormalisation and factorisation scales is given explicitly in [28].

[3] The quantities $\Delta\sigma_{\mu+}$ and $\Delta\sigma_{\mu-}$ are positive for all choices of top mass and scales we have considered.

'central' PDF set, a number of other sets (usually 40 for the CTEQ family PDFs, 30 for the MRST family ones) and properly combining their differences. According to the CTEQ and MRST Collaborations, the resulting uncertainty should roughly represent a 90% confidence level. We have chosen to follow the prescription by Nadolsky and Sullivan [29], and determine asymmetric uncertainties in the form

$$\Delta\sigma_{\mathrm{PDF+}} = \sqrt{\sum_i \left(\max \left[\sigma(set_{+i}) - \sigma(set_0), \sigma(set_{-i}) - \sigma(set_0), 0 \right] \right)^2} \,, \qquad (7)$$

$$\Delta\sigma_{\mathrm{PDF-}} = \sqrt{\sum_i \left(\max \left[\sigma(set_0) - \sigma(set_{+i}), \sigma(set_0) - \sigma(set_{-i}), 0 \right] \right)^2} \,. \qquad (8)$$

where all cross sections are evaluated with

$$\xi_{\mathrm{F}} = 1 \,, \qquad \xi_{\mathrm{R}} = 1 \,. \qquad (9)$$

In eqs. (7) and (8), set_0 represents the central set, and the sums run over all pairs of PDFs in the given PDF error set. For each pair, we denote by set_{+i} and set_{-i} the positive and negative displacement member of the pair. We also introduce the symbols

$$\mathrm{PDFs+} \quad = \quad \sigma(\mathrm{central}) + \Delta\sigma_{\mathrm{PDF+}} \qquad (10)$$

$$\mathrm{PDFs-} \quad = \quad \sigma(\mathrm{central}) - \Delta\sigma_{\mathrm{PDF-}} \qquad (11)$$

which we shall use in the following.

To facilitate the determination of theoretical cross section corresponding to mass values different than the current best fit, we provide our results in the form of the coefficients of the parametrization

$$\sigma(m_t) = A + B(m_t - 171) + C(m_t - 171)^2 + D(m_t - 171)^3 \,. \qquad (12)$$

The parameters were fitted to the exact results in the range $150 \leq m_t \leq 190$ GeV, with a precision of the order of 1-2 per mille. The A coefficient has been fixed equal to the cross section at $m_t = 171$ GeV. The fit parameters are given in table 1 and table 3, for the Tevatron and the LHC respectively[4].

For each PDF set we have listed separately the 'central' value (scales = m_t, central PDF set) and the maximum and the minimum found by varying the scales according to the above procedure and evaluating the asymmetric PDFs uncertainties. The main effects of the different PDFs and of the uncertainties can of course be read off directly from the A coefficient, which corresponds to the $t\bar{t}$ cross section evaluated at $m_t = 171$ GeV. The display of results obtained with many different PDF sets, both very recent and older, is meant to allow for an easy estimate of the variation (or lack thereof) of the cross section predictions as a consequence of evolving parton distribution functions sets.

We summarise here what might be considered our "best" predictions for $t\bar{t}$ production at the LHC, at $m_t = 171$ GeV:

$$\sigma_{t\bar{t}}^{\mathrm{NLO+NLL}}(\mathrm{LHC}, m_t = 171 \text{ GeV}, \mathrm{CTEQ6.5}) = 908 \ _{-85(9.3\%)}^{+82(9.0\%)} \text{ (scales) } _{-29(3.2\%)}^{+30(3.3\%)} \text{ (PDFs) pb} \qquad (13)$$

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[4]Since the LHC is scheduled to run in 2008 at a centre of mass energy of 10 TeV, we have also provided predictions for this energy in table 2, in the case a measurement of the total cross section of top production should prove possible.

Tevatron, $p\bar{p}$ at $\sqrt{s} = 1960$ GeV		A (pb)	B (pb/GeV)	C (pb/GeV^2)	D (pb/GeV^3)
CTEQ6M	Central	7.59	-0.237	4.39 $\times 10^{-3}$	-6.32 $\times 10^{-5}$
	Scales+	7.89	-0.247	4.60 $\times 10^{-3}$	-6.66 $\times 10^{-5}$
	Scales−	7.07	-0.221	4.11 $\times 10^{-3}$	-5.92 $\times 10^{-5}$
	PDFs+	8.26	-0.260	4.86 $\times 10^{-3}$	-7.02 $\times 10^{-5}$
	PDFs−	7.12	-0.222	4.08 $\times 10^{-3}$	-5.82 $\times 10^{-5}$
CTEQ6.1	Central	7.77	-0.244	4.53 $\times 10^{-3}$	-6.51 $\times 10^{-5}$
	Scales+	8.08	-0.254	4.74 $\times 10^{-3}$	-6.86 $\times 10^{-5}$
	Scales−	7.23	-0.227	4.23 $\times 10^{-3}$	-6.09 $\times 10^{-5}$
	PDFs+	8.53	-0.269	5.04 $\times 10^{-3}$	-7.27 $\times 10^{-5}$
	PDFs−	7.20	-0.224	4.12 $\times 10^{-3}$	-5.87 $\times 10^{-5}$
CTEQ6.5	Central	7.61	-0.237	4.38 $\times 10^{-3}$	-6.28 $\times 10^{-5}$
	Scales+	7.90	-0.247	4.58 $\times 10^{-3}$	-6.61 $\times 10^{-5}$
	Scales−	7.08	-0.221	4.10 $\times 10^{-3}$	-5.89 $\times 10^{-5}$
	PDFs+	8.14	-0.256	4.78 $\times 10^{-3}$	-6.91 $\times 10^{-5}$
	PDFs−	7.24	-0.224	4.11 $\times 10^{-3}$	-5.85 $\times 10^{-5}$
CTEQ6.6	Central	7.48	-0.233	4.32 $\times 10^{-3}$	-6.20 $\times 10^{-5}$
	Scales +	7.77	-0.243	4.52 $\times 10^{-3}$	-6.53 $\times 10^{-5}$
	Scales −	6.96	-0.218	4.04 $\times 10^{-3}$	-5.80 $\times 10^{-5}$
	PDFs +	7.99	-0.251	4.70 $\times 10^{-3}$	-6.79 $\times 10^{-5}$
	PDFs −	7.09	-0.220	4.02 $\times 10^{-3}$	-5.72 $\times 10^{-5}$
MRST2001E	Central	7.66	-0.242	4.53 $\times 10^{-3}$	-6.60 $\times 10^{-5}$
	Scales+	7.97	-0.252	4.75 $\times 10^{-3}$	-6.98 $\times 10^{-5}$
	Scales−	7.13	-0.225	4.24 $\times 10^{-3}$	-6.17 $\times 10^{-5}$
	PDFs+	7.94	-0.252	4.75 $\times 10^{-3}$	-6.95 $\times 10^{-5}$
	PDFs−	7.44	-0.233	4.35 $\times 10^{-3}$	-6.31 $\times 10^{-5}$
MRST2004nlo	Central	7.99	-0.253	4.77 $\times 10^{-3}$	-6.95 $\times 10^{-5}$
MRST2006nnlo	Central	7.93	-0.253	4.76 $\times 10^{-3}$	-6.92 $\times 10^{-5}$
	Scales+	8.27	-0.264	5.00 $\times 10^{-3}$	-7.33 $\times 10^{-5}$
	Scales−	7.37	-0.235	4.44 $\times 10^{-3}$	-6.45 $\times 10^{-5}$
	PDFs+	8.17	-0.261	4.93 $\times 10^{-3}$	-7.19 $\times 10^{-5}$
	PDFs−	7.73	-0.245	4.61 $\times 10^{-3}$	-6.68 $\times 10^{-5}$

Table 1: Coefficients of the parametrization $\sigma(m_t) = A + B(m_t - 171) + C(m_t - 171)^2 + D(m_t - 171)^3$ for the NLO+NLL $t\bar{t}$ cross section (picobarn) at the Tevatron, for various PDF sets. The fit must not be used outside the range $150 \leq m_t \leq 190$ GeV. The quantities Scales± and PDFs± are defined in eqs. (5), (6), (10), and (11).

LHC, pp at $\sqrt{s} = 10$ TeV		A (pb)	B (pb/GeV)	C (pb/GeV^2)	D (pb/GeV^3)
CTEQ6M	Central	425	-12.1	0.211	-2.89 $\times 10^{-3}$
	Scales+	462	-13.2	0.232	-3.20 $\times 10^{-3}$
	Scales−	386	-10.9	0.189	-2.58 $\times 10^{-3}$
	PDFs+	445	-12.5	0.216	-2.94 $\times 10^{-3}$
	PDFs−	406	-11.7	0.205	-2.82 $\times 10^{-3}$
CTEQ6.1	Central	428	-12.1	0.211	-2.87 $\times 10^{-3}$
	Scales+	465	-13.2	0.232	-3.19 $\times 10^{-3}$
	Scales−	389	-10.9	0.189	-2.57 $\times 10^{-3}$
	PDFs+	450	-12.5	0.216	-2.93 $\times 10^{-3}$
	PDFs−	406	-11.6	0.205	-2.81 $\times 10^{-3}$
CTEQ6.5	Central	414	-11.7	0.205	-2.79 $\times 10^{-3}$
	Scales+	450	-12.9	0.226	-3.09 $\times 10^{-3}$
	Scales−	376	-10.6	0.184	-2.50 $\times 10^{-3}$
	PDFs+	434	-12.2	0.211	-2.85 $\times 10^{-3}$
	PDFs−	396	-11.3	0.199	-2.72 $\times 10^{-3}$
CTEQ6.6	Central	414	-11.8	0.206	-2.81 $\times 10^{-3}$
	Scales +	451	-12.9	0.227	-3.12 $\times 10^{-3}$
	Scales −	376	-10.6	0.185	-2.51 $\times 10^{-3}$
	PDFs +	433	-12.2	0.211	-2.86 $\times 10^{-3}$
	PDFs −	396	-11.4	0.200	-2.75 $\times 10^{-3}$
MRST2001E	Central	446	-12.6	0.217	-2.94 $\times 10^{-3}$
	Scales+	486	-13.8	0.240	-3.27 $\times 10^{-3}$
	Scales−	405	-11.3	0.195	-2.63 $\times 10^{-3}$
	PDFs+	457	-12.8	0.220	-2.97 $\times 10^{-3}$
	PDFs−	439	-12.4	0.216	-2.92 $\times 10^{-3}$
MRST2004nlo	Central	455	-12.8	0.221	-2.99 $\times 10^{-3}$
MRST2006nnlo	Central	446	-12.5	0.216	-2.92 $\times 10^{-3}$
	Scales+	486	-13.7	0.238	-3.24 $\times 10^{-3}$
	Scales−	404	-11.3	0.194	-2.60 $\times 10^{-3}$
	PDFs+	454	-12.7	0.218	-2.93 $\times 10^{-3}$
	PDFs−	438	-12.3	0.214	-2.89 $\times 10^{-3}$

Table 2: Coefficients of the parametrization $\sigma(m_t) = A + B(m_t - 171) + C(m_t - 171)^2 + D(m_t - 171)^3$ for the NLO+NLL $t\bar{t}$ cross section (picobarn) at the LHC with $\sqrt{s} = 10$ TeV, for various PDF sets. The fit must not be used outside the range $150 \leq m_t \leq 190$ GeV. The quantities Scales$\pm$ and PDFs$\pm$ are defined in eqs. (5), (6), (10), and (11).

LHC, pp at $\sqrt{s} = 14$ TeV		A (pb)	B (pb/GeV)	C (pb/GeV^2)	D (pb/GeV^3)
CTEQ6M	Central	933	-25.3	0.423	-5.60 $\times 10^{-3}$
	Scales+	1018	-27.7	0.468	-6.22 $\times 10^{-3}$
	Scales−	846	-22.8	0.379	-4.99 $\times 10^{-3}$
	PDFs+	962	-25.8	0.432	-5.73 $\times 10^{-3}$
	PDFs−	903	-24.6	0.413	-5.44 $\times 10^{-3}$
CTEQ6.1	Central	934	-25.2	0.421	-5.56 $\times 10^{-3}$
	Scales+	1019	-27.7	0.466	-6.19 $\times 10^{-3}$
	Scales−	847	-22.7	0.377	-4.95 $\times 10^{-3}$
	PDFs+	965	-25.8	0.430	-5.70 $\times 10^{-3}$
	PDFs−	902	-24.5	0.411	-5.40 $\times 10^{-3}$
CTEQ6.5	Central	908	-24.5	0.411	-5.46 $\times 10^{-3}$
	Scales+	990	-26.9	0.455	-6.08 $\times 10^{-3}$
	Scales−	823	-22.1	0.368	-4.87 $\times 10^{-3}$
	PDFs+	938	-25.2	0.420	-5.57 $\times 10^{-3}$
	PDFs−	879	-23.9	0.401	-5.29 $\times 10^{-3}$
CTEQ6.6	Central	911	-24.7	0.413	-5.47 $\times 10^{-3}$
	Scales +	993	-27.1	0.457	-6.09 $\times 10^{-3}$
	Scales −	826	-22.2	0.370	-4.87 $\times 10^{-3}$
	PDFs +	939	-25.2	0.422	-5.58 $\times 10^{-3}$
	PDFs −	881	-24.0	0.404	-5.36 $\times 10^{-3}$
MRST2001E	Central	965	-25.9	0.429	-5.63 $\times 10^{-3}$
	Scales+	1054	-28.4	0.475	-6.27 $\times 10^{-3}$
	Scales−	874	-23.3	0.384	-5.00 $\times 10^{-3}$
	PDFs+	981	-26.2	0.434	-5.68 $\times 10^{-3}$
	PDFs−	954	-25.6	0.426	-5.57 $\times 10^{-3}$
MRST2004nlo	Central	982	-26.3	0.436	-5.72 $\times 10^{-3}$
MRST2006nnlo	Central	961	-25.7	0.426	-5.58 $\times 10^{-3}$
	Scales+	1050	-28.3	0.472	-6.21 $\times 10^{-3}$
	Scales−	870	-23.1	0.381	-4.96 $\times 10^{-3}$
	PDFs+	972	-25.9	0.428	-5.62 $\times 10^{-3}$
	PDFs−	949	-25.4	0.422	-5.53 $\times 10^{-3}$

Table 3: Coefficients of the parametrization $\sigma(m_t) = A + B(m_t - 171) + C(m_t - 171)^2 + D(m_t - 171)^3$ for the NLO+NLL $t\bar{t}$ cross section (picobarn) at the LHC, for various PDF sets. The fit must not be used outside the range $150 \leq m_t \leq 190$ GeV. The quantities Scales± and PDFs± are defined in eqs. (5), (6), (10), and (11).

$$\sigma_{t\bar{t}}^{\text{NLO+NLL}}(\text{LHC}, m_t = 171 \text{ GeV}, \text{MRST2006nnlo}) = 961 \, {}^{+89(9.2\%)}_{-91(9.4\%)} \text{ (scales)} \, {}^{+11(1.1\%)}_{-12(1.2\%)} \text{ (PDFs)} \text{ pb}$$
(14)

Note that we quote separately the results for MRST and CTEQ, since they are not fully consistent. For reference, we also give here the pure fixed-order results (i.e., without including threshold resummation) at the NLO and the LO

$$\sigma_{t\bar{t}}^{\text{NLO}}(\text{LHC}, m_t = 171 \text{ GeV}, \text{CTEQ6.5}) = 875 \, {}^{+102(11.6\%)}_{-100(11.5\%)} \text{ (scales)} \, {}^{+30(3.4\%)}_{-29(3.3\%)} \text{ (PDFs)} \text{ pb}$$
(15)

$$\sigma_{t\bar{t}}^{\text{LO}}(\text{LHC}, m_t = 171 \text{ GeV}, \text{CTEQ6.5}) = 583 \, {}^{+165(28.2\%)}_{-120(20.7\%)} \text{ (scales)} \, {}^{+20(3.4\%)}_{-19(3.3\%)} \text{ (PDFs)} \text{ pb}$$
(16)

$$\sigma_{t\bar{t}}^{\text{NLO}}(\text{LHC}, m_t = 171 \text{ GeV}, \text{MRST2006nnlo}) = 927 \, {}^{+109(11.7\%)}_{-107(11.5\%)} \text{ (scales)} \, {}^{+11(1.2\%)}_{-12(1.3\%)} \text{ (PDFs)} \text{ pb}$$
(17)

$$\sigma_{t\bar{t}}^{\text{LO}}(\text{LHC}, m_t = 171 \text{ GeV}, \text{MRST2006nnlo}) = 616 \, {}^{+172(27.9\%)}_{-126(20.5\%)} \text{ (scales)} \, {}^{+7.3(1.2\%)}_{-7.8(1.3\%)} \text{ (PDFs)} \text{ pb}$$
(18)

We have decided not to combine the scales and PDFs uncertainties into a single error. The reason for refraining from doing so is that the scales uncertainty (and, to some extent, probably also the PDFs one) is not fully characterised in statistical terms. As a consequence, additional hypotheses will be needed in order to combine the two uncertainties into a single probability density function for the resulting cross section, with well defined confidence levels. Further discussions on the interplay between scales and PDFs uncertainties can be found in Appendix A.

We finally present our "best" predictions for $t\bar{t}$ production at the Tevatron, at $m_t = 171$ GeV:

$$\sigma_{t\bar{t}}^{\text{NLO+NLL}}(\text{Tev}, m_t = 171 \text{ GeV}, \text{CTEQ6.5}) = 7.61 \, {}^{+0.30(3.9\%)}_{-0.53(6.9\%)} \text{ (scales)} \, {}^{+0.53(7\%)}_{-0.36(4.8\%)} \text{ (PDFs)} \text{ pb}$$
(19)

$$\sigma_{t\bar{t}}^{\text{NLO+NLL}}(\text{Tev}, m_t = 171 \text{ GeV}, \text{MRST2006nnlo}) = 7.93 \, {}^{+0.34(4.3\%)}_{-0.56(7.1\%)} \text{ (scales)} \, {}^{+0.24(3.1\%)}_{-0.20(2.5\%)} \text{ (PDFs)} \text{ pb}.$$
(20)

As done for the LHC in eqs. (15)–(18), we also report the NLO and LO results:

$$\sigma_{t\bar{t}}^{\text{NLO}}(\text{Tev}, m_t = 171 \text{ GeV}, \text{CTEQ6.5}) = 7.35 \, {}^{+0.38(5.1\%)}_{-0.80(10.9\%)} \text{ (scales)} \, {}^{+0.49(6.6\%)}_{-0.34(4.6\%)} \text{ (PDFs)} \text{ pb}$$
(21)

$$\sigma_{t\bar{t}}^{\text{LO}}(\text{Tev}, m_t = 171 \text{ GeV}, \text{CTEQ6.5}) = 5.92 \, {}^{+2.34(39.5\%)}_{-1.54(26.1\%)} \text{ (scales)} \, {}^{+0.32(5.5\%)}_{-0.24(4.1\%)} \text{ (PDFs)} \text{ pb}$$
(22)

$$\sigma_{t\bar{t}}^{\text{NLO}}(\text{Tev}, m_t = 171 \text{ GeV}, \text{MRST2006nnlo}) = 7.62 \, {}^{+0.45(5.9\%)}_{-0.88(11.6\%)} \text{ (scales)} \, {}^{+0.23(3\%)}_{-0.18(2.4\%)} \text{ (PDFs)} \text{ pb}$$
(23)

$$\sigma_{t\bar{t}}^{\text{LO}}(\text{Tev}, m_t = 171 \text{ GeV}, \text{MRST2006nnlo}) = 6.05 \, {}^{+2.47(40.8\%)}_{-1.61(26.6\%)} \text{ (scales)} \, {}^{+0.16(2.6\%)}_{-0.13(2.1\%)} \text{ (PDFs)} \text{ pb}.$$
(24)

2 Discussion of the $t\bar{t}$ cross section results at the LHC

For all parameter choices we have considered, the scales uncertainties affecting the $t\bar{t}$ cross section at the LHC are much larger than those due to PDFs. In this section, we therefore focus on exploring the effect of the NLL resummation on the cross section. We can do so by comparing the NLO prediction at $m_t = 171$ GeV, and its uncertainty due to scales variations as described above, in the

NLO and NLO+NLL approximations respectively. We find that the 'central' prediction is increased by less than 4%, and the scales uncertainty is only very mildly affected, going from $\pm 11.5\%$ in the NLO case to $\pm 9\%$ in the NLO+NLL one. This points to a relatively minor impact of threshold resummation on the LHC cross section, as expected as a consequence of the relatively large distance of the $t\bar{t}$ production threshold from the LHC centre of mass energy (for comparison, the uncertainty at the Tevatron is almost halved when going from NLO to NLO+NLL). One should also note that, again contrary to the Tevatron case, exploring *independent* scale variations has a non-negligible effect: keeping $\mu_R = \mu_F$ (as done e.g. in ref. [27]) would result in an uncertainty estimate for the NLO+NLL case of only $^{+7}_{-5}\%$. We remind the reader that the fact PDF fits are performed with $\mu_R = \mu_F$ does not force us to use $\mu_R = \mu_F$ in the cross section. In fact an independent variation of the two scales in our matched calculation leads to variations in the result that are beyond the NLO+NLL approximation. It is thus legitimate to add this independent variation to the sources of uncertainties. It then turns out that the $\mu_R \neq \mu_F$ approach leads to a much larger variation. We thus conclude that there may be accidental cancellation in the scale variation when one keeps $\mu_R = \mu_F$, leading to an unreliably small estimate of the error.

Another important element in the assessment of the systematics related to the resummation is the estimate of the impact of beyond-NLL corrections. To parametrize these corrections, a constant A was introduced in [13] (where more details about its role are given):

$$C_{ij} \rightarrow C_{ij} \left(1 - \frac{A}{N + A - 1} \right), \quad ij = q\bar{q}, gg \; . \tag{25}$$

C_{ij} here represents the N-independent term of the Mellin transform of the NLO partonic cross section. The replacement in the previous equation gives vanishing first moments, is irrelevant for large N, and does not introduce poles on the real N axis. Different choices of A give rise to different resummed cross sections, all consistent with each other at the NLL level and NNLL level[5]. They therefore parametrize the possible exponentiation of finite, non-logarithmic terms appearing at orders higher than NLO. It was noticed already in [13] that the choice $A = 0$ was leading to a scale dependence typically a factor of two smaller than was obtained with $A = 2$. For the sake of being conservative, we therefore selected $A = 2$ in our subsequent phenomenological analysis [8], as well as in the results presented in the previous section. We would like to reiterate here this observation, by showing the results that we would have obtained at the LHC if we had chosen $A = 0$. In the case of the CTEQ6.5 PDF, and $m_t = 171$ GeV, the scale dependence obtained by varying μ_F and μ_R independently is:

$$\sigma_{t\bar{t}}^{\text{NLO+NLL}(A=0)}(\text{LHC}, m_t = 171 \text{ GeV}, \text{CTEQ6.5}) = 945 \, ^{+95(10\%)}_{-85(9.0\%)} \; (\text{scales}) \quad \text{pb} \qquad [\mu_F \neq \mu_R] \; , \tag{26}$$

which is approximately 5% larger than the value obtained with $A = 2$, a variation consistent with the estimated uncertainty. If we had chosen to set $\mu_F = \mu_R$, the result would have been:

$$\sigma_{t\bar{t}}^{\text{NLO+NLL}(A=0)}(\text{LHC}, m_t = 171 \text{ GeV}, \text{CTEQ6.5}) = 945 \, ^{+19(2\%)}_{-7(0.7\%)} \; (\text{scales}) \quad \text{pb} \qquad [\mu_F = \mu_R] \; . \tag{27}$$

Notice that the combination of $A = 0$ and $\mu_R = \mu_F$ leads to a dramatic reduction of the uncertainty. In particular, the reduction is significant also with respect to the $A = 2$, $\mu_R = \mu_F$ case, discussed above. The choice $A = 0$ and $\mu_R = \mu_F$ is what was used in the recent analysis of the resummed

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[5]Changing A corresponds to vary terms suppressed by powers of N, which cannot be determined within the soft gluon approximation.

NNLL cross section of [23], leading to a similar uncertainty, at the 2% level. We conclude that it is not as yet clear whether the improvement found in [23] is a genuine reduction of the uncertainty, that would survive the independent variation of μ_{R} and μ_{F} and with the introduction of a parameter similar to our A.

We also note that the resummed NLO+NLL results quoted in ref. [23] differ from ours to the extent of a few percent. We have checked that this is not due to the choice made in [23] of limiting the contribution of NLL resummation to a $t\bar{t}$ mass range near the production threshold, and using only the NLO result above such range (contrary to what stated in [23], we do not perform the matching in this way, but follow instead the procedure detailed in [13]). Rather, the small discrepancy is due to the misleading way in which eq. (25) is presented in [13], and in which it was accordingly interpreted and used in [23]. There is in fact a mismatch in the Mellin N argument appearing in the expression of coefficient C_{ij} used in eqs. (54) and (58) of [13], with $N+1$ being the correct argument of the resummation function, rather than N. The numerical implementation of these relations, in [13] and in this paper, are nevertheless consistent, and equivalent to a shift $N \to N+1$ in (25). An erratum to clarify this issue has been submitted for [13].

Another issue we wish to comment on is the relative size of the scale and the PDFs uncertainty. It was observed in [8] that the latter was important, and almost dominant, at the Tevatron. This appears not to be the case anymore at the LHC: again according to the procedure described above, at $m_t = 171$ GeV we find uncertainties of the order of $\pm 3\%$ with CTEQ6.5 and $\pm 1.5\%$ with MRST2006nnlo and MRST2001E. The main reason for this improvement is that at the LHC the range of x values for the partons relevant to top production is much smaller than at the Tevatron, and falls in a region where the experimental knowledge of both quark and gluon PDFs is much better constrained by data. It is worth noting that the central results given by these two PDF sets, differing by about 6%, are not fully compatible, despite (or because of) the apparently very small estimated uncertainty. We also point out the MRST and CTEQ use different conventions for the Tolerance parameter; the consequence of this is that, had the two collaborations followed exactly the same fitting procedure, the PDF uncertainty resulting from using an MRST family set would still be a factor of about $\sqrt{2}$ smaller than that obtained with a CTEQ set.

While the PDFs uncertainty is probably still somewhat underestimated, as shown by the partially conflicting central values of CTEQ6.5 and MRST2006nnlo, it is probably safe to conclude that, the very interesting progress with the NNLL resummation [23] notwithstanding, a definitive assessment of our understanding of the $t\bar{t}$ cross section at the LHC will have to wait for the full, massive NNLO calculation.

3   Very heavy fermion production

We now present production rates for a pair of fermions (belonging to the fundamental representation of $SU(3)_{colour}$) heavier than top, using the same computations described for the $t\bar{t}$ cross sections. We shall generically denote such fermion pair by $T\bar{T}$. These particles arise naturally in BSM theories with strongly-coupled dynamics; they can be of different species, which can for example be classified according to their transformation properties under SU(2)$_{\mathrm{L}}\otimes$SU(2)$_{\mathrm{R}}\otimes$U(1). As far as pair production is concerned, however, these details are largely irrelevant, since this process is expected to be dominated by QCD effects, and this is the reason why we can apply our NLO+NLL, NLO, and LO results to the computations of $T\bar{T}$ rates. In doing so, we shall neglect possible contributions

m_T	NLO+NLL	NLO	LO
0.5	$4006^{+232(5.8\%)\ +466(11.7\%)}_{-276(6.9\%)\ -332(8.3\%)}$	$3802^{+342(9\%)\ +455(12.0\%)}_{-421(11.1\%)\ -322(8.5\%)}$	$2726^{+876(32.1\%)\ +314(11.5\%)}_{-618(22.7\%)\ -221(8.1\%)}$
0.6	$1429^{+76.3(5.3\%)\ +195(13.7\%)}_{-93.2(6.5\%)\ -134(9.4\%)}$	$1352^{+116(8.6\%)\ +188(14.0\%)}_{-148(11.0\%)\ -129(9.5\%)}$	$980.7^{+319(32.5\%)\ +130(13.3\%)}_{-225(22.9\%)\ -88.7(9.1\%)}$
0.7	$577.6^{+28.7(5\%)\ +89.0(15.4\%)}_{-36.0(6.2\%)\ -59.6(10.3\%)}$	$545.1^{+44.9(8.2\%)\ +85.5(15.7\%)}_{-59.3(10.9\%)\ -57.0(10.5\%)}$	$399.1^{+131(32.9\%)\ +59.0(14.8\%)}_{-92.1(23.1\%)\ -39.2(9.8\%)}$
0.8	$256.0^{+12.0(4.7\%)\ +43.4(17\%)}_{-15.4(6\%)\ -28.4(11.1\%)}$	$241.0^{+19.3(8\%)\ +41.4(17.2\%)}_{-26.1(10.8\%)\ -27.0(11.2\%)}$	$177.8^{+58.9(33.1\%)\ +28.5(16.1\%)}_{-41.3(23.2\%)\ -18.6(10.5\%)}$
0.9	$121.7^{+5.41(4.4\%)\ +22.3(18.4\%)}_{-7.06(5.8\%)\ -14.4(11.8\%)}$	$114.3^{+8.95(7.8\%)\ +21.2(18.5\%)}_{-12.3(10.8\%)\ -13.5(11.9\%)}$	$84.71^{+28.3(33.4\%)\ +14.6(17.2\%)}_{-19.8(23.4\%)\ -9.32(11\%)}$
1.0	$61.12^{+2.59(4.2\%)\ +12.0(19.6\%)}_{-3.45(5.6\%)\ -7.58(12.4\%)}$	$57.25^{+4.42(7.7\%)\ +11.3(19.7\%)}_{-6.17(10.8\%)\ -7.11(12.4\%)}$	$42.57^{+14.3(33.7\%)\ +7.75(18.2\%)}_{-10.0(23.5\%)\ -4.88(11.5\%)}$
1.1	$32.05^{+1.32(4.1\%)\ +6.68(20.8\%)}_{-1.76(5.5\%)\ -4.15(13\%)}$	$29.94^{+2.29(7.7\%)\ +6.25(20.9\%)}_{-3.24(10.8\%)\ -3.88(12.9\%)}$	$22.31^{+7.56(33.9\%)\ +4.28(19.2\%)}_{-5.28(23.6\%)\ -2.66(11.9\%)}$
1.2	$17.41^{+0.706(4.1\%)\ +3.83(22\%)}_{-0.939(5.4\%)\ -2.35(13.5\%)}$	$16.23^{+1.24(7.6\%)\ +3.57(22\%)}_{-1.76(10.9\%)\ -2.18(13.4\%)}$	$12.10^{+4.13(34.2\%)\ +2.43(20.1\%)}_{-2.88(23.8\%)\ -1.50(12.4\%)}$
1.3	$9.737^{+0.388(4\%)\ +2.25(23.2\%)}_{-0.516(5.3\%)\ -1.36(14\%)}$	$9.049^{+0.693(7.7\%)\ +2.09(23.1\%)}_{-0.989(10.9\%)\ -1.26(13.9\%)}$	$6.745^{+2.32(34.4\%)\ +1.42(21.1\%)}_{-1.61(23.9\%)\ -0.864(12.8\%)}$
1.4	$5.578^{+0.218(3.9\%)\ +1.36(24.3\%)}_{-0.291(5.2\%)\ -0.810(14.5\%)}$	$5.169^{+0.398(7.7\%)\ +1.25(24.2\%)}_{-0.569(11\%)\ -0.745(14.4\%)}$	$3.848^{+1.34(34.7\%)\ +0.850(22.1\%)}_{-0.927(24.1\%)\ -0.511(13.3\%)}$
1.5	$3.260^{+0.126(3.9\%)\ +0.833(25.5\%)}_{-0.168(5.2\%)\ -0.492(15.1\%)}$	$3.012^{+0.235(7.8\%)\ +0.763(25.3\%)}_{-0.335(11.1\%)\ -0.450(14.9\%)}$	$2.238^{+0.783(35\%)\ +0.518(23.1\%)}_{-0.543(24.2\%)\ -0.309(13.8\%)}$
1.6	$1.938^{+0.074(3.8\%)\ +0.520(26.8\%)}_{-0.099(5.1\%)\ -0.304(15.7\%)}$	$1.785^{+0.141(7.9\%)\ +0.474(26.5\%)}_{-0.200(11.2\%)\ -0.277(15.5\%)}$	$1.323^{+0.467(35.3\%)\ +0.321(24.3\%)}_{-0.323(24.4\%)\ -0.190(14.4\%)}$
1.7	$1.169^{+0.044(3.7\%)\ +0.329(28.2\%)}_{-0.059(5.1\%)\ -0.191(16.3\%)}$	$1.073^{+0.086(8\%)\ +0.299(27.8\%)}_{-0.122(11.4\%)\ -0.173(16.1\%)}$	$0.793^{+0.282(35.6\%)\ +0.202(25.5\%)}_{-0.195(24.6\%)\ -0.119(15\%)}$
1.8	$0.714^{+0.026(3.7\%)\ +0.212(29.6\%)}_{-0.036(5\%)\ -0.123(17.2\%)}$	$0.653^{+0.053(8.2\%)\ +0.191(29.2\%)}_{-0.075(11.5\%)\ -0.109(16.8\%)}$	$0.480^{+0.173(35.9\%)\ +0.129(26.9\%)}_{-0.119(24.7\%)\ -0.075(15.7\%)}$
1.9	$0.440^{+0.016(3.7\%)\ +0.137(31.2\%)}_{-0.022(5\%)\ -0.078(17.7\%)}$	$0.401^{+0.033(8.4\%)\ +0.123(30.8\%)}_{-0.047(11.7\%)\ -0.070(17.5\%)}$	$0.294^{+0.107(36.3\%)\ +0.083(28.3\%)}_{-0.073(24.9\%)\ -0.048(16.4\%)}$
2.0	$0.274^{+0.010(3.6\%)\ +0.090(32.9\%)}_{-0.013(5\%)\ -0.051(18.6\%)}$	$0.248^{+0.021(8.5\%)\ +0.080(32.4\%)}_{-0.029(11.8\%)\ -0.045(18.3\%)}$	$0.181^{+0.066(36.6\%)\ +0.054(30\%)}_{-0.045(25.1\%)\ -0.031(17.3\%)}$

Table 4: Cross sections (in fb) for the production of $T\bar{T}$ pairs at the LHC, computed with CTEQ6.5 PDFs. The mass of the heavy fermion T is expressed in TeV. For each entry of the table, we give the central value of the cross section, with the scale and PDF uncertainties.

of non-SM intermediate states resulting from e.g. a $q\bar{q}$ annihilation. Our aim is therefore not that of providing a complete phenomenological study of $T\bar{T}$ cross sections at the LHC, but rather that of assessing the scale and PDF uncertainties affecting the QCD contribution to the production of heavy fermion pairs. In what follow, we shall consider the mass range 0.5 TeV$\leq m_T \leq$ 2 TeV for the heavy fermion.

Our results are presented in tables 4 and 5. A few comments are in order.

- The scale dependence of the NLO+NLL cross section is small for all values of m_T, and decreases monotonically with increasing m_T. This is to be expected, since the heavier the fermion, the closer the kinematics is to the threshold.

- The scale dependence of the NLO cross section starts by decreasing, but then tends to increase with increasing m_T. This is the signal of the necessity of including threshold corrections. On the other hand, the scale dependence of the LO cross section is always extremely large. This fact must be taken into proper account if an estimate of the K factor is needed. In particular, one must precisely understand which hard scale is used in a LO computation (e.g. in a standard parton-shower Monte Carlo).

- The relative PDF uncertainty is extremely large in the case of CTEQ6.5. When MRST2006nnlo

11

m_T	NLO+NLL	NLO	LO
0.5	$4462^{+267.4(6\%)}_{-314.0(7\%)}{}^{+197.4(4.4\%)}_{-172.6(3.9\%)}$	$4236^{+392.9(9.3\%)}_{-480.5(11.3\%)}{}^{+191.9(4.5\%)}_{-167.7(4\%)}$	$3017^{+988.3(32.8\%)}_{-694.6(23\%)}{}^{+132.0(4.4\%)}_{-115.9(3.8\%)}$
0.6	$1599^{+88.7(5.5\%)}_{-107.0(6.7\%)}{}^{+81.2(5.1\%)}_{-70.1(4.4\%)}$	$1513^{+134.9(8.9\%)}_{-170.9(11.3\%)}{}^{+78.2(5.2\%)}_{-67.5(4.5\%)}$	$1089^{+362.9(33.3\%)}_{-253.8(23.3\%)}{}^{+53.8(4.9\%)}_{-46.6(4.3\%)}$
0.7	$648.8^{+33.7(5.2\%)}_{-41.6(6.4\%)}{}^{+36.3(5.6\%)}_{-31.0(4.8\%)}$	$611.9^{+52.9(8.6\%)}_{-68.9(11.3\%)}{}^{+34.7(5.7\%)}_{-29.6(4.8\%)}$	$444.0^{+145.0(33.8\%)}_{-104.5(23.5\%)}{}^{+23.9(5.4\%)}_{-20.4(4.6\%)}$
0.8	$288.2^{+14.1(4.9\%)}_{-17.8(6.2\%)}{}^{+17.3(6\%)}_{-14.6(5.1\%)}$	$271.0^{+22.9(8.4\%)}_{-30.5(11.2\%)}{}^{+16.4(6.1\%)}_{-13.9(5.1\%)}$	$198.0^{+67.5(34.1\%)}_{-46.9(23.7\%)}{}^{+11.4(5.7\%)}_{-9.57(4.8\%)}$
0.9	$137.2^{+6.42(4.7\%)}_{-8.21(6\%)}{}^{+8.72(6.4\%)}_{-7.28(5.3\%)}$	$128.6^{+10.7(8.3\%)}_{-14.5(11.3\%)}{}^{+8.23(6.4\%)}_{-6.85(5.3\%)}$	$94.34^{+32.5(34.4\%)}_{-22.5(23.9\%)}{}^{+5.74(6.1\%)}_{-4.72(5\%)}$
1.0	$68.97^{+3.09(4.5\%)}_{-4.02(5.8\%)}{}^{+4.61(6.7\%)}_{-3.77(5.5\%)}$	$64.48^{+5.30(8.2\%)}_{-7.27(11.3\%)}{}^{+4.32(6.7\%)}_{-3.52(5.5\%)}$	$47.43^{+16.5(34.8\%)}_{-11.4(24\%)}{}^{+3.05(6.4\%)}_{-2.44(5.1\%)}$
1.1	$36.21^{+1.56(4.3\%)}_{-2.06(5.7\%)}{}^{+2.53(7\%)}_{-2.03(5.6\%)}$	$33.75^{+2.76(8.2\%)}_{-3.82(11.3\%)}{}^{+2.37(7\%)}_{-1.88(5.6\%)}$	$24.87^{+8.72(35\%)}_{-6.02(24.2\%)}{}^{+1.69(6.8\%)}_{-1.31(5.3\%)}$
1.2	$19.69^{+0.824(4.2\%)}_{-1.10(5.6\%)}{}^{+1.44(7.3\%)}_{-1.12(5.7\%)}$	$18.30^{+1.49(8.2\%)}_{-2.08(11.4\%)}{}^{+1.35(7.4\%)}_{-1.04(5.7\%)}$	$13.50^{+4.77(35.3\%)}_{-3.29(24.4\%)}{}^{+0.987(7.3\%)}_{-0.734(5.4\%)}$
1.3	$11.03^{+0.449(4.1\%)}_{-0.604(5.5\%)}{}^{+0.857(7.8\%)}_{-0.642(5.8\%)}$	$10.22^{+0.837(8.2\%)}_{-1.17(11.5\%)}{}^{+0.802(7.9\%)}_{-0.593(5.8\%)}$	$7.533^{+2.68(35.6\%)}_{-1.85(24.5\%)}{}^{+0.597(7.9\%)}_{-0.425(5.6\%)}$
1.4	$6.324^{+0.252(4\%)}_{-0.341(5.4\%)}{}^{+0.524(8.3\%)}_{-0.377(6\%)}$	$5.843^{+0.482(8.3\%)}_{-0.674(11.5\%)}{}^{+0.492(8.4\%)}_{-0.348(6\%)}$	$4.303^{+1.55(35.9\%)}_{-1.06(24.7\%)}{}^{+0.372(8.7\%)}_{-0.253(5.9\%)}$
1.5	$3.702^{+0.145(3.9\%)}_{-0.197(5.3\%)}{}^{+0.329(8.9\%)}_{-0.227(6.1\%)}$	$3.409^{+0.284(8.3\%)}_{-0.397(11.6\%)}{}^{+0.310(9.1\%)}_{-0.210(6.2\%)}$	$2.506^{+0.908(36.2\%)}_{-0.622(24.8\%)}{}^{+0.238(9.5\%)}_{-0.155(6.2\%)}$
1.6	$2.203^{+0.085(3.8\%)}_{-0.116(5.3\%)}{}^{+0.211(9.6\%)}_{-0.140(6.3\%)}$	$2.022^{+0.171(8.5\%)}_{-0.238(11.8\%)}{}^{+0.199(9.9\%)}_{-0.130(6.4\%)}$	$1.483^{+0.542(36.5\%)}_{-0.370(25\%)}{}^{+0.155(10.4\%)}_{-0.097(6.5\%)}$
1.7	$1.330^{+0.051(3.8\%)}_{-0.069(5.2\%)}{}^{+0.138(10.4\%)}_{-0.088(6.6\%)}$	$1.217^{+0.104(8.6\%)}_{-0.145(11.9\%)}{}^{+0.131(10.7\%)}_{-0.082(6.7\%)}$	$0.890^{+0.328(36.8\%)}_{-0.224(25.1\%)}{}^{+0.102(11.5\%)}_{-0.061(6.9\%)}$
1.8	$0.813^{+0.031(3.8\%)}_{-0.042(5.2\%)}{}^{+0.091(11.2\%)}_{-0.056(6.9\%)}$	$0.741^{+0.065(8.8\%)}_{-0.089(12.1\%)}{}^{+0.087(11.7\%)}_{-0.052(7\%)}$	$0.540^{+0.201(37.2\%)}_{-0.137(25.3\%)}{}^{+0.068(12.6\%)}_{-0.039(7.3\%)}$
1.9	$0.502^{+0.019(3.7\%)}_{-0.026(5.1\%)}{}^{+0.061(12.2\%)}_{-0.036(7.2\%)}$	$0.455^{+0.041(8.9\%)}_{-0.056(12.2\%)}{}^{+0.058(12.8\%)}_{-0.034(7.4\%)}$	$0.330^{+0.124(37.5\%)}_{-0.084(25.5\%)}{}^{+0.046(13.8\%)}_{-0.026(7.8\%)}$
2.0	$0.312^{+0.012(3.7\%)}_{-0.016(5.1\%)}{}^{+0.041(13.2\%)}_{-0.023(7.5\%)}$	$0.282^{+0.026(9.1\%)}_{-0.035(12.4\%)}{}^{+0.039(13.9\%)}_{-0.022(7.8\%)}$	$0.204^{+0.077(37.9\%)}_{-0.052(25.7\%)}{}^{+0.031(15\%)}_{-0.017(8.2\%)}$

Table 5: Cross sections (in fb) for the production of $T\bar{T}$ pairs at the LHC, computed with MRST2006nnlo PDFs. The mass of the heavy fermion T is expressed in TeV. For each entry of the table, we give the central value of the cross section, with the scale and PDF uncertainties.

sets are used, that uncertainty is smaller by a factor of about 2–3, consistently with what already observed in the case of top production. By and large, the PDF uncertainty affects equally the NLO+NLL, the NLO, and the LO cross sections. At the largest m_T values considered here, it prevents one from giving a precise prediction even in the case of the NLO+NLL computation. It will therefore be highly desirable to measure the PDFs at the LHC for intermediate- and large-x regions, using e.g. low-mass final states produced at large rapidity.

4   Conclusions

In this paper we have produced and tabulated updated predictions for the next-to-leading order plus next-to-leading log resummed (NLO+NLL) cross sections for $t\bar{t}$ production at the Tevatron and at the LHC. QCD cross sections for heavy fermion production at the LHC are also given.

The theoretical uncertainties due to unknown higher orders and to the imperfect knowledge of the parton distribution function sets are explored in detail and also tabulated. NLO and LO results are also given, for reference and comparison.

The main results for $t\bar{t}$ production at the LHC ($\sqrt{S} = 14$ TeV) are the following:

$$\sigma_{t\bar{t}}^{\mathrm{NLO+NLL}}(\mathrm{LHC}, m_t = 171 \ \mathrm{GeV}, \mathrm{CTEQ6.5}) = 908 \ ^{+82(9.0\%)}_{-85(9.3\%)} \ (\mathrm{scales}) \ ^{+30(3.3\%)}_{-29(3.2\%)} \ (\mathrm{PDFs}) \ \ \mathrm{pb} \qquad (28)$$

$$\sigma_{t\bar{t}}^{\mathrm{NLO+NLL}}(\mathrm{LHC}, m_t = 171 \ \mathrm{GeV}, \mathrm{MRST2006nnlo}) = 961 \ ^{+89(9.2\%)}_{-91(9.4\%)} \ (\mathrm{scales}) \ ^{+11(1.1\%)}_{-12(1.2\%)} \ (\mathrm{PDFs}) \ \ \mathrm{pb} \qquad (29)$$

Cross sections obtained with two of the most recent PDF sets are given, since they appear to be only partially compatible within their respective uncertainties.

Finally, we note that ref. [23] recently produced an approximated NNLO cross section by truncating a soft-gluon NNLL resummed calculation to order α_s^4. Their phenomenological analysis produces cross sections for the LHC with extremely small scales uncertainty, of order 2-3%, sensibly smaller than ours. We have argued in section. 2 that such a small uncertainty also arises at the NLO+NLL level by requiring the factorization and renormalization scales to be equal. It will therefore be interesting to verify whether such reduced scale dependence found in [23] survives a test with independent scales, thus showing a genuine improvement due to the added NNLO terms, or whether it is an intrinsic consequence of keeping the scales equal.

Note added

After this work was completed, a new study of the $t\bar{t}$ cross section at the Tevatron and LHC appeared in ref. [30].

Acknowledgements

A On scale and PDF uncertainties

The definitions we have adopted in eqs. (3) and (4) for scale of uncertainty, and in eqs. (7) and (8) for PDF uncertainty, are by no means unique. In this section, we shall briefly illustrate other choices.

The possibility of making different choices stems from the observation that scale and PDF uncertainties have to be combined in order to obtain an estimate of the overall uncertainty affecting the cross section. The way in which this combination is to be performed is at present unclear, given the fact that neither the scale uncertainty nor the PDF uncertainty (the latter owing to the fact that PDF error sets are derived in violation of the $\Delta\chi^2 = 1$ rule) follow the laws of statistical errors.

Scale uncertainty can in general be written as

$$\Delta\sigma_{\mu+} = \sigma\left(\xi_F^{(max)}, \xi_R^{(max)}\right) - \sigma(1,1) \,, \tag{30}$$

$$\Delta\sigma_{\mu-} = \sigma(1,1) - \sigma\left(\xi_F^{(min)}, \xi_R^{(min)}\right) \,, \tag{31}$$

where different prescriptions can be devised for the determination of $(\xi_F^{(max)}, \xi_R^{(max)})$ and of $(\xi_F^{(min)}, \xi_R^{(min)})$. As far as PDF uncertainty is concerned, one always makes use of

$$\delta\sigma_{\mathrm{PDF+}}(\xi_F, \xi_R) = \sqrt{\sum_i \left(\max\left[\sigma(set_{+i}) - \sigma(set_0), \sigma(set_{-i}) - \sigma(set_0), 0\right]\right)^2} \,, \tag{32}$$

$$\delta\sigma_{\mathrm{PDF-}}(\xi_F, \xi_R) = \sqrt{\sum_i \left(\max\left[\sigma(set_0) - \sigma(set_{+i}), \sigma(set_0) - \sigma(set_{-i}), 0\right]\right)^2} \,, \tag{33}$$

and then defines

$$\Delta\sigma_{\mathrm{PDF+}} = \delta\sigma_{\mathrm{PDF+}}\left(\bar{\xi}_F^{(max)}, \bar{\xi}_R^{(max)}\right) \,, \tag{34}$$

$$\Delta\sigma_{\mathrm{PDF-}} = \delta\sigma_{\mathrm{PDF-}}\left(\bar{\xi}_F^{(min)}, \bar{\xi}_R^{(min)}\right) \,, \tag{35}$$

where again the values of $(\bar{\xi}_F^{(max)}, \bar{\xi}_R^{(max)})$ and of $(\bar{\xi}_F^{(min)}, \bar{\xi}_R^{(min)})$ at which the r.h.s. of these equations are evaluated are a matter of choice.

We limit ourselves to give four examples.

A) Our default choice, illustrated in sect. 1 and which gives rise to the results presented in this paper, is equivalent to solving

$$\max_{\{\xi_F, \xi_R\}} \left[\sigma(\xi_F, \xi_R) - \sigma(1,1)\right] = \sigma\left(\xi_F^{(max)}, \xi_R^{(max)}\right) - \sigma(1,1) \,, \tag{36}$$

$$\min_{\{\xi_F, \xi_R\}} \left[\sigma(\xi_F, \xi_R) - \sigma(1,1)\right] = \sigma\left(\xi_F^{(min)}, \xi_R^{(min)}\right) - \sigma(1,1) \,, \tag{37}$$

for $(\xi_F^{(max)}, \xi_R^{(max)})$ and $(\xi_F^{(min)}, \xi_R^{(min)})$, which are then used in eqs. (30) and (31). The PDF uncertainty is defined by setting

$$\left(\bar{\xi}_F^{(max)}, \bar{\xi}_R^{(max)}\right) = (1,1) \,, \qquad \left(\bar{\xi}_F^{(min)}, \bar{\xi}_R^{(min)}\right) = (1,1) \,. \tag{38}$$

14

B) The scale uncertainty is defined in the same way as done in item *A)*. For the PDF uncertainty, we set

$$\left(\bar{\xi}_{\mathrm{F}}^{(max)},\bar{\xi}_{\mathrm{R}}^{(max)}\right) = \left(\xi_{\mathrm{F}}^{(max)},\xi_{\mathrm{R}}^{(max)}\right) , \qquad \left(\bar{\xi}_{\mathrm{F}}^{(min)},\bar{\xi}_{\mathrm{R}}^{(min)}\right) = \left(\xi_{\mathrm{F}}^{(min)},\xi_{\mathrm{R}}^{(min)}\right) . \tag{39}$$

with $(\xi_{\mathrm{F}}^{(max)},\xi_{\mathrm{R}}^{(max)})$ and $(\xi_{\mathrm{F}}^{(min)},\xi_{\mathrm{R}}^{(min)})$ computed again as in eqs. (36) and (37).

C) We first solve

$$\max_{\{\xi_{\mathrm{F}},\xi_{\mathrm{R}}\}}\left[\sigma(\xi_{\mathrm{F}},\xi_{\mathrm{R}}) + \delta\sigma_{\mathrm{PDF+}}(\xi_{\mathrm{F}},\xi_{\mathrm{R}}) - \sigma(1,1)\right] =$$
$$\sigma\left(\xi_{\mathrm{F}}^{(max)},\xi_{\mathrm{R}}^{(max)}\right) + \delta\sigma_{\mathrm{PDF+}}\left(\xi_{\mathrm{F}}^{(max)},\xi_{\mathrm{R}}^{(max)}\right) - \sigma(1,1) , \tag{40}$$

$$\min_{\{\xi_{\mathrm{F}},\xi_{\mathrm{R}}\}}\left[\sigma(\xi_{\mathrm{F}},\xi_{\mathrm{R}}) - \delta\sigma_{\mathrm{PDF-}}(\xi_{\mathrm{F}},\xi_{\mathrm{R}}) - \sigma(1,1)\right] =$$
$$\sigma\left(\xi_{\mathrm{F}}^{(min)},\xi_{\mathrm{R}}^{(min)}\right) - \delta\sigma_{\mathrm{PDF-}}\left(\xi_{\mathrm{F}}^{(min)},\xi_{\mathrm{R}}^{(min)}\right) - \sigma(1,1) , \tag{41}$$

for $(\xi_{\mathrm{F}}^{(max)},\xi_{\mathrm{R}}^{(max)})$ and $(\xi_{\mathrm{F}}^{(min)},\xi_{\mathrm{R}}^{(min)})$. These values are then used to determine the scale uncertainty according to eqs. (3) and (4), and the PDF uncertainty by setting

$$\left(\bar{\xi}_{\mathrm{F}}^{(max)},\bar{\xi}_{\mathrm{R}}^{(max)}\right) = \left(\xi_{\mathrm{F}}^{(max)},\xi_{\mathrm{R}}^{(max)}\right) , \qquad \left(\bar{\xi}_{\mathrm{F}}^{(min)},\bar{\xi}_{\mathrm{R}}^{(min)}\right) = \left(\xi_{\mathrm{F}}^{(min)},\xi_{\mathrm{R}}^{(min)}\right) , \tag{42}$$

which are then used in eqs (34) and (35).

D) The scale uncertainty is defined in the same way as done in item *A)*. For PDF uncertainty, we solve

$$\max_{\{\xi_{\mathrm{F}},\xi_{\mathrm{R}}\}}\left[\delta\sigma_{\mathrm{PDF+}}(\xi_{\mathrm{F}},\xi_{\mathrm{R}})\right] = \delta\sigma_{\mathrm{PDF+}}\left(\bar{\xi}_{\mathrm{F}}^{(max)},\bar{\xi}_{\mathrm{R}}^{(max)}\right) , \tag{43}$$

$$\max_{\{\xi_{\mathrm{F}},\xi_{\mathrm{R}}\}}\left[\delta\sigma_{\mathrm{PDF-}}(\xi_{\mathrm{F}},\xi_{\mathrm{R}})\right] = \delta\sigma_{\mathrm{PDF-}}\left(\bar{\xi}_{\mathrm{F}}^{(min)},\bar{\xi}_{\mathrm{R}}^{(min)}\right) , \tag{44}$$

for $(\bar{\xi}_{\mathrm{F}}^{(max)},\bar{\xi}_{\mathrm{R}}^{(max)})$ and $(\bar{\xi}_{\mathrm{F}}^{(min)},\bar{\xi}_{\mathrm{R}}^{(min)})$, which are then used in eqs (34) and (35).

Items *B)-D)* follow the same logic, namely finding the absolute maximum and minimum of the cross section, by various combinations of scale and PDF uncertainties. In this sense, it is not fully justified to quote these two uncertainties separately, although it is still convenient for bookkeeping. These approaches stem from the observation that, in a hadroproduction QCD computation, unknown higher orders also enter the determination of the PDFs, and one is therefore entitled to use the full information on the PDF uncertainty in the determination of the scale dependence. In fact, the three methods give very similar results, with *D)* being the most conservative, i.e. resulting in the largest overall cross section uncertainty.

On the other hand, by following the procedure outlined in item *A)*, one is able to better assess the separate dependence upon scales and PDFs. It should be observed that, while the quantities $\delta\sigma_{\mathrm{PDF\pm}}(\xi_{\mathrm{F}},\xi_{\mathrm{R}})$ depend on ξ_{F} and ξ_{R} roughly in the same way as the cross sections $\sigma(\xi_{\mathrm{F}},\xi_{\mathrm{R}})$, the ratios

$$\delta\sigma_{\mathrm{PDF\pm}}(\xi_{\mathrm{F}},\xi_{\mathrm{R}})\Big/\sigma(\xi_{\mathrm{F}},\xi_{\mathrm{R}}) \tag{45}$$

are extremely stable with respect to variations of ξ_{F} and ξ_{R}. This implies that the *relative* PDF uncertainty on the central value of the cross section, that is

$$\Delta\sigma_{\mathrm{PDF}\pm}\Big/\sigma(1,1) \tag{46}$$

is basically identical to any of those in eq. (45), if one follows item *A)*. This is not the case for items *B)–D)*; the relative uncertainty due to $\Delta\sigma_{\mathrm{PDF}+}$ ($\Delta\sigma_{\mathrm{PDF}-}$) tends to be larger (smaller) than that computed according to item *A)*.

The consideration above led us to prefer the procedure of item *A)* for the computation of the results presented in this paper. This has also the advantage that it renders the calculation less demanding from the point of view of CPU time. We conclude by stressing that for top production the procedures of items *B)–D)* would have given similar results as that of item *A)*.

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